A Generalized Net Model of the Abdominal Aorta and Its Branches as a Part of the Vascular System

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Abstract. In a series of papers, Generalized Net (GN) models of the ways of functioning of the different systems and organs in the human body are described in general. Each GN model of a particular system or organ can be detailed and made more complex. In this paper a GN model of the abdominal aorta and its branches of the vascular system is proposed.

Keywords: Generalized nets, Modelling, Abdominal aorta, Vascular system.

1 Introduction

The heart together with the blood vessels – arteries, veins and lymph vessels – form the Cardiovascular System (CVS) [10, 13, 26]. The life of every human being begins with the first contraction (systole) of the heart and ends with the last. The main function of the CVS is to transport the blood which is the base of the exchange processes of the organism. Despite the fact that blood is a complex colloid and the blood vessels are much more different in terms of structure and properties than the ordinary water-pipes, to a vast degree the blood circulation is subject to the laws of hydrodynamics, but with some specific features. Because of this, the blood circulation in the blood vessels is defined as hemodynamics. The heart resembles a pump with an infinite displacement because the system is closed and inside the system a certain quantity of blood circulates but with different quality and functions – arterial or venous. As a result of the exchange processes in the tissues, the arterial blood becomes venous and the venous blood becomes arterial by absorbing the blood from the liver (nutrients) and the oxygen in the lungs (releasing carbon dioxide at the same time).

For practical purposes, the blood circulation in the arterial system is of special importance because the problems arising here are the most dangerous and oftentimes lead to irreparable consequences and death. The venous and lymph systems have to a greater degree collective and draining functions, and the diseases, connected to them, in most cases have chronic character and only in some cases emergency interventions are required. Therefore, with the aim of avoiding the unnecessary complication, our efforts are aimed at the construction of a mathematical model only of the arterial system. The venous system will be included only schematically with the aim of completing the cycle of the blood circulation and making sense of the significance of the heart as a main force behind the processes of the CVS.

The CVS is a complex system working thanks to many fine mechanisms for regulation and adaptation according to the inner environment of the organism and the outer environment. Certain laws exist both at the level of separate organs, and at the organism level, which determine the correct and effective function of the CVS. Apart from this, the complexity and diversity of the CVS pathology should be taken into account. The gained empiric experience, the scientific discoveries in this field, the numerous medicaments and invasive technologies allow for a relatively effective treatment. However, medical science is still far away from the complete understanding of this complex system.

Modelling the CVS gives the possibilities to evaluate its parameters and to analyze its behavior and functioning [12, 14, 28]. In the present paper, for the construction of the model the theory of the Generalized Nets (GNs, see[1]) is used. A lot of GN models of the different systems and organs in the human body are described in [2, 4–9, 15–25]. Most of them were collected in the book [3]. In this paper, a GN model of the abdominal aorta and its branches as a part of the vascular system is constructed. Using hierarchical operators from the GN theory, it can be used as a subnet of the GN model of the vascular system presented in [3]. The anatomical segmentation of the aorta as a main artery of the human body is important, because these segments are analyzed separately in the context of diagnosing aortic disease [27]. The abdominal aorta begins at the level of the diaphragm [26, 10, 11]. The main arteries arising from the abdominal aorta are the common and internal iliac arteries, the common and superficial femoral arteries, the anterior and posterior tibial arteries, and the peronial artery.

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The GN-model contains 24 transitions and 50 places. The following types of tokens are used:

- α-tokens – they represent the blood in the arteries and capillaries, i.e., before the metabolism;

- β-tokens – they represent the blood in the veins, i.e., after the metabolism;
- χ-token – it represents the heart;
- μ_1-, μ_2-, ..., μ_{11}-tokens – they represent the human body matter in which metabolism processes flow as follows:
  - μ_1-token represents the abdominal aorta;
  - μ_2- and μ_3-tokens represent the left and the right internal iliac artery;
  - μ_4- and μ_5-tokens represent the left and the right profunda femoris artery;
  - μ_6- and μ_9-tokens represent the left and the right peronial artery;
  - μ_7- and μ_{10}-tokens represent the left and the right left anterior artery;
  - μ_8- and μ_{11}-tokens represent the left and the right posterior artery.

  Each one of these tokens obtain as a characteristic:

  "blood, quantity, time-moment".

The transitions of the GN represent the following functions:

- Z_1 – the functions of the heart;
- Z_2 – the functions of the abdominal aorta;
- Z_3 and Z_4 – the functions of the left/right common iliac artery;
- Z_5 and Z_6 – the functions of the left/right internal iliac artery;
- Z_8 and Z_9 – the functions of the left/right common femoral artery;
- Z_{11} and Z_{12} – the functions of the left/right profunda femoris artery;
- Z_{14} and Z_{15} – the functions of the left/right superficial femoral artery;
- Z_{17} and Z_{20} – the functions of the left/right peronial artery;
- Z_{18} and Z_{21} – the functions of the left/right anterior tibial artery;
- Z_{19} and Z_{22} – the functions of the left/right posterior tibial artery;
- $Z_7, Z_{10}, Z_{13}, Z_{16}, Z_{23}, Z_{24}$ – the functions of the veins after metabolism.

Below is a formal description of the GN-transitions.

$$Z_1 = \langle \{l_2, l_3\}, \{l_1, l_2\},$$

$$\begin{array}{c|cc} & l_1 & l_2 \\ \hline l_2 & true & true \\ l_3 & false & true \end{array} \rangle.$$

At each time-step, token from place l_3 enters place l_2 and unites with token χ and simultaneously, token χ splits to two tokens - the same token χ and a token α that enters place l_1.

$$Z_2 = \langle \{l_1, l_6, l_{15}, {}_{20}\}, \{l_3, l_4, l_5, l_6\},$$

$$\begin{array}{c|cccc} & l_3 & l_4 & l_5 & l_6 \\ \hline l_1 & false & true & true & true \\ l_6 & true & false & false & true \\ l_{15} & false & false & false & true \\ l_{20} & false & false & false & true \end{array} \rangle.$$

At each time-step, token α from place l_1 splits to three α-tokens: the first of them enters place l_6 and unites with token μ_1 that obtains the above mentioned characteristic; the other two α-tokens enter places l_4 and l_5. In the same time-moment, token μ_1 splits to two tokens - the same token μ_1 and a token β that enters place l_3, where it unites with the β-tokens from places l_{15} and l_{20}.

$$Z_3 = \langle \{l_4\}, \{l_7, l_8\},$$

$$\begin{array}{c|cc} & l_7 & l_8 \\ \hline l_4 & true & true \end{array}\rangle.$$

At each time-step, token from place l_4 splits to two α-tokens that enter places l_7 and l_8.

$$Z_4 = \langle \{l_5\}, \{l_9, l_{10}\},$$

$$\begin{array}{c|cc} & l_9 & l_{10} \\ \hline l_5 & true & true \end{array}\rangle.$$

At each time-step, token from place l_5 splits to two α-tokens that enter places l_9 and l_{10}.

$$Z_5 = \langle \{l_7, l_{11}\}, \{l_{11}, l_{12}\},$$

$$\begin{array}{c|cc} & l_{11} & l_{12} \\ \hline l_7 & true & false \\ l_{11} & true & true \end{array}\rangle.$$

At each time-step, the α-token from place l_7 enters place l_{11} and unites with token μ_2 and simultaneously, token μ_2 splits to two tokens - the same token μ_2 and a token β that enters place l_{12}.

$$Z_6 = \langle \{l_{10}, l_{14}\}, \{l_{13}, l_{14}\},$$

$$\begin{array}{c|cc} & l_{13} & l_{14} \\ \hline l_{10} & false & true \\ l_{14} & true & true \end{array}\rangle.$$

At each time-step, the α-token from place l_{10} enters place l_{14} and unites with token μ_2 and simultaneously, token μ_2 splits to two tokens - the same token μ_2 and a token β that enters place l_{13}.

$$Z_7 = \langle \{l_{12}, l_{25}\}, \{l_{15}\},$$

$$\begin{array}{c|c} & l_{15} \\ \hline l_{12} & true \\ l_{25} & true \end{array}\rangle.$$

At each time-step, the β-tokens from places l_{12} and l_{25} enter place l_{15} where they unite in a β-token.

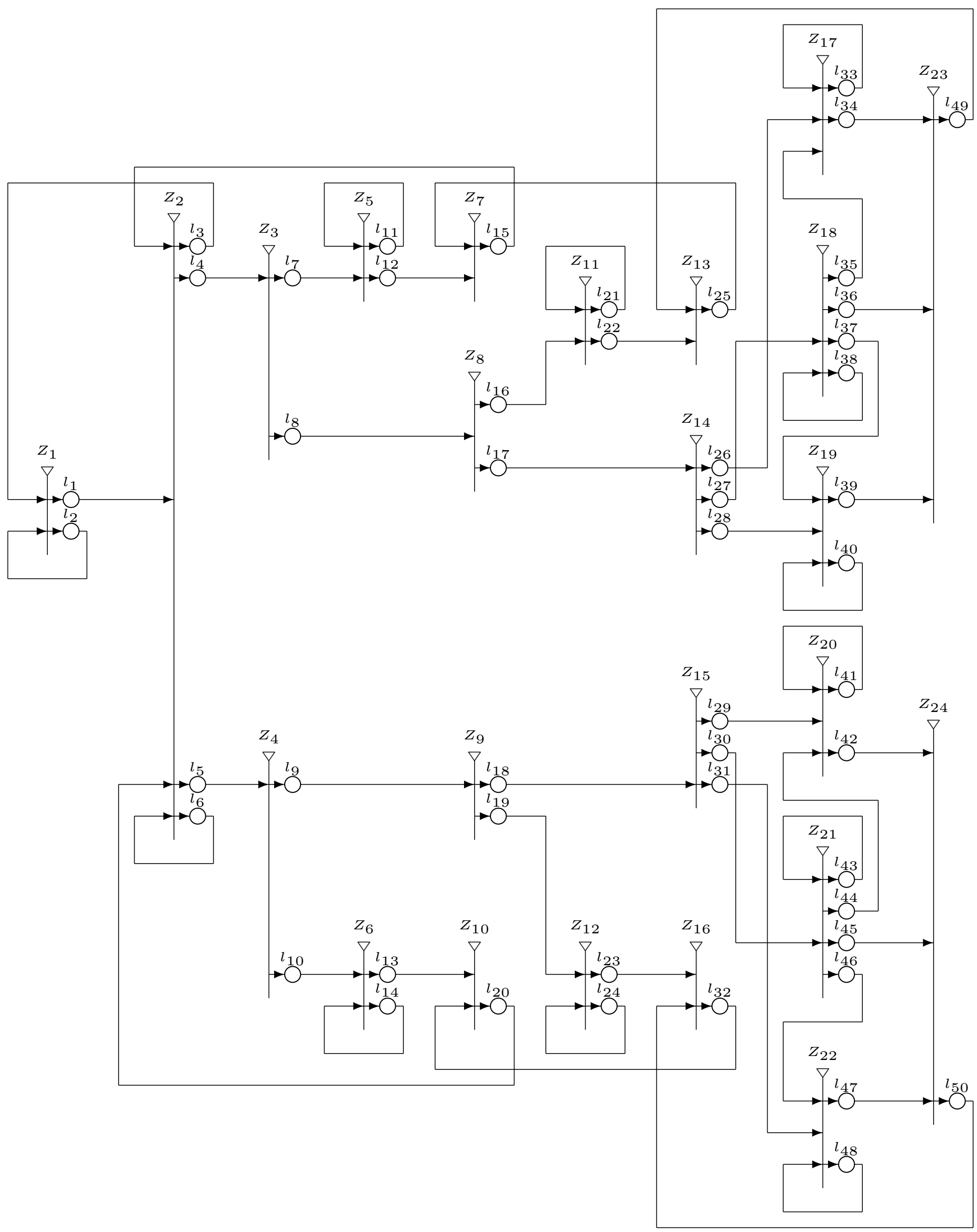

Fig. 1. Graphical representation of the GN model of the abdominal aorta and its branches as a part of the vascular system.

$$Z_8 = \langle \{l_8\}, \{l_{16}, l_{17}\},$$

$$\begin{array}{c|cc} & l_{16} & l_{17} \\ \hline l_8 & true & true \end{array}\rangle.$$

At each time-step, the α-token from place l_8 splits to two α-tokens that enter places l_{16} and l_{17}.

$$Z_9 = \langle \{l_9\}, \{l_{18}, l_{19}\},$$

$$\begin{array}{c|cc} & l_{18} & l_{19} \\ \hline l_9 & true & true \end{array}\rangle.$$

At each time-step, the α-token from place l_9 splits to two α-tokens that enter places l_{18} and l_{19}.

$$Z_{10} = \langle \{l_{13}, l_{32}\}, \{l_{20}\},$$

$$\begin{array}{c|c} & l_{20} \\ \hline l_{13} & true \\ l_{32} & true \end{array}\rangle.$$

At each time-step, the β-tokens from places l_{13} and l_{32} enter place l_{20} where they unite in a β-token.

$$Z_{11} = \langle \{l_{16}, l_{21}\}, \{l_{21}, l_{22}\},$$

$$\begin{array}{c|cc} & l_{21} & l_{22} \\ \hline l_{16} & true & false \\ l_{21} & true & true \end{array}\rangle.$$

At each time-step, the α-token from place l_{16} enters place l_{21} and unites with token μ_4 and simultaneously, token μ_4 splits to two tokens - the same token μ_4 and a token β that enters place l_{22}.

$$Z_{12} = \langle \{l_{19}, l_{24}\}, \{l_{23}, l_{24}\},$$

$$\begin{array}{c|cc} & l_{23} & l_{24} \\ \hline l_{19} & false & true \\ l_{24} & true & true \end{array}\rangle.$$

At each time-step, the α-token from place l_{19} enters place l_{24} and unites with token μ_5 and simultaneously, token μ_5 splits to two tokens - the same token μ_5 and a token β that enters place l_{23}.

$$Z_{13} = \langle \{l_{22}, l_{49}\}, \{l_{25}\},$$

$$\begin{array}{c|c} & l_{25} \\ \hline l_{22} & true \\ l_{49} & true \end{array}\rangle.$$

At each time-step, the β-tokens from places l_{22} and l_{49} enter place l_{25} where they unite in a β-token.

$$Z_{14} = \langle \{l_{17}\}, \{l_{26}, l_{27}, l_{28}\},$$
$$\begin{array}{c|ccc} & l_{26} & l_{27} & l_{28} \\ \hline l_{17} & true & true & true \end{array} \rangle.$$

At each time-step, token α from place l_{17} splits to three α-tokens that enter places l_{26}, l_{27} and l_{28}.

$$Z_{15} = \langle \{l_{18}\}, \{l_{29}, l_{30}, l_{31}\},$$
$$\begin{array}{c|ccc} & l_{29} & l_{30} & l_{31} \\ \hline l_{18} & true & true & true \end{array} \rangle.$$

At each time-step, token α from place l_{18} splits to three α-tokens that enter places l_{29}, l_{30} and l_{31}.

$$Z_{16} = \langle \{l_{23}, l_{50}\}, \{l_{32}\},$$
$$\begin{array}{c|c} & l_{32} \\ \hline l_{23} & true \\ l_{50} & true \end{array} \rangle.$$

At each time-step, the β-tokens from places l_{23} and l_{50} enter place l_{32} where they unite in a β-token.

$$Z_{17} = \langle \{l_{26}, l_{33}, l_{35}\}, \{l_{33}, l_{34}\},$$
$$\begin{array}{c|cc} & l_{33} & l_{34} \\ \hline l_{26} & true & false \\ l_{33} & true & true \\ l_{35} & true & false \end{array} \rangle.$$

At each time-step, the α-token from place l_{26} enters place l_{33} and unites with token μ_6 and simultaneously, token μ_6 splits to two tokens - the same token μ_6 and a β-token that enters place l_{34}. When there is an α-token in place l_{35}, it enters place l_{33} and unites with token μ_6.

$$Z_{18} = \langle \{l_{27}, l_{38}\}, \{l_{35}, l_{36}, l_{37}, l_{38}\},$$
$$\begin{array}{c|cccc} & l_{35} & l_{36} & l_{37} & l_{38} \\ \hline l_{27} & W_{27,35} & false & W_{27,37} & true \\ l_{38} & false & true & false & true \end{array} \rangle,$$

where
$W_{27,35}$ = "there is necessity for blood for the left peronial artery ",
$W_{27,37}$ = "there is necessity for blood for the left posterior tibial artery".

At each time-step, the α-token from place l_{27} enters place l_{38} and unites with token μ_7 and simultaneously, token μ_7 splits to two tokens - the same token μ_7

and a β-token that enters place l_{36}. When $W_{27,35} = true$ and/or $W_{27,37} = true$, the α-token from place l_{27} splits to one of two additional α-tokens that enter places l_{35} and/or l_{37}, respectively, depending on the truth-values of predicates $W_{27,35}$ and $W_{27,37}$.

$$Z_{19} = \langle \{l_{28}, l_{37}, l_{40}\}, \{l_{39}, l_{40}\},$$

$$\begin{array}{c|cc} & l_{39} & l_{40} \\ \hline l_{28} & false & true \\ l_{40} & true & true \\ l_{37} & false & true \end{array} \rangle.$$

At each time-step, the α-token from place l_{28} enters place l_{40} and unites with token μ_8 and simultaneously, token μ_8 splits to two tokens - the same token μ_8 and a β-token that enters place l_{39}. When there is an α-token in place l_{37}, it enters place l_{40} and unites with token μ_8.

$$Z_{20} = \langle \{l_{29}, l_{41}, l_{44}\}, \{l_{41}, l_{42}\},$$

$$\begin{array}{c|cc} & l_{41} & l_{42} \\ \hline l_{29} & true & false \\ l_{41} & true & true \\ l_{44} & true & false \end{array} \rangle.$$

At each time-step, the α-token from place l_{29} enters place l_{41} and unites with token μ_9 and simultaneously, token μ_9 splits to two tokens - the same token μ_9 and a β-token that enters place l_{42}. When there is an α-token in place l_{44}, it enters place l_{41} and unites with token μ_9.

$$Z_{21} = \langle \{l_{30}, l_{43}\}, \{l_{43}, l_{44}, l_{45}, l_{46}\},$$

$$\begin{array}{c|cccc} & l_{43} & l_{44} & l_{45} & l_{46} \\ \hline l_{30} & true & W_{30,44} & false & W_{30,46} \\ l_{43} & true & false & true & false \end{array} \rangle,$$

where
$W_{30,44}$ = "there is necessity for blood for the right peronial artery. ",
$W_{30,46}$ = "there is necessity for blood for the right posterior tibial artery".

At each time-step, the α-token from place l_{30} enters place l_{43} and unites with token μ_{10} and simultaneously, token μ_{10} splits to two tokens - the same token μ_{10} and a β-token that enters place l_{45}. When $W_{30,44} = true$ and/or $W_{30,46} = true$, the α-token from place l_{30} splits to one of two additional α-tokens that enter places l_{44} and/or l_{46}, respectively, in respect of the truth-values of predicates $W_{30,44}$ and $W_{30,46}$.

$$Z_{22} = \langle \{l_{31}, l_{46}, l_{48}\}, \{l_{47}, l_{48}\},$$

$$\begin{array}{c|cc} & l_{47} & l_{48} \\ \hline l_{31} & false & true \\ l_{48} & true & true \\ l_{46} & false & true \end{array}\rangle.$$

At each time-step, the α-token from place l_{31} enters place l_{48} and unites with token μ_{11} and simultaneously, token μ_{11} splits to two tokens - the same token μ_{11} and a β-token that enters place l_{47}. When there is an α-token in place l_{46}, it enters place l_{48} and unites with token μ_{11}.

$$Z_{23} = \langle \{l_{34}, l_{36}, l_{39}\}, \{l_{49}\},$$

$$\begin{array}{c|c} & l_{49} \\ \hline l_{34} & true \\ l_{36} & true \\ l_{39} & true \end{array}\rangle.$$

At each time-step, the β-tokens from places l_{34}, l_{36} and l_{39} enter place l_{49} where they unite in a β-token.

$$Z_{24} = \langle \{l_{42}, l_{45}, l_{47}\}, \{l_{50}\},$$

$$\begin{array}{c|c} & l_{50} \\ \hline l_{42} & true \\ l_{45} & true \\ l_{47} & true \end{array}\rangle.$$

At each time-step, the β-tokens from places l_{42}, l_{45} and l_{47} enter place l_{50} where they unite in a β-token.

3 Conclusion

In the present paper, a GN model of the abdominal aorta and its branches as a part of the vascular system is presented. The purpose of the construction of the GN model of the CVS is to complement and enrich the knowledge about the CVS, or at least about a part of it. Objective criteria and parameters can be chosen which through the GN model would give us a clearer understanding about the state of the CVS, presence and weight of the pathology, the risk degree and a relatively correct prognosis for the development of one disease or another.

In our future research, we intend to construct a GN model of the aortic arch branches of the CVS as a continuation of our work on the modeling of systems and organs of the human body with GNs.

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